Monotone Boolean Functions of Sum-Free Sets ⋆

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Abstract. We investigate sum-free subsets of $[n]$ from the perspective of monotone Boolean functions. Since every subset of a sum-free set is itself sum-free, the family of sum-free sets forms a downward-closed system and can be represented by a monotone Boolean function. Restricting attention to the classical condition that no element is the sum of two others, we analyze the corresponding vertices of the Boolean cube B^n. We determine the highest layer of B^n containing vertices associated with sum-free sets and show that it is $\lfloor n/2 \rfloor + 1$. Furthermore, we characterize and enumerate all non-sum-free triples of $[n]$. This yields an enumeration of all sum-free triples. Together, these results establish a structural correspondence between sum-free sets and monotone Boolean functions. Applications include distributed database partitioning, frequency assignment in wireless networks, image mining, association rule mining, and a variety of other domains, especially those involving combinatorial optimization and large-scale data analysis.

Keywords: Monotone Boolean functions · Hypercube · Sum-free sets · Data mining applications.

1 Introduction

Monotone Boolean functions (MBF) and sum-free sets (SFS) are two independent actors in extremal combinatorics ([1], [2], [4], [5], [6], [7], [8], [10], [11], [12], [16], [18], [19], [20]). At the same time, they share an interesting structural similarity. A monotone Boolean function never decreases when any of its variables are changed from 0 to 1, whereas a sum-free set remains sum-free when restricted to any of its subsets. Exploring this commonality is one of the main objectives of our research.

Let us start with introducing some of the basic applications.

Frequency Assignment in Wireless Networks, and distributed database partitioning.

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Frequency Hopping in Cellular and Wireless Networks. In code-division multiple access (CDMA) and frequency-hopping spread-spectrum (FHSS) systems, transmitters switch among different carrier frequencies to avoid interference. By selecting frequency channels from a sum-free set, it is possible to ensure that the sum (or difference) of any two utilized frequencies does not coincide with a third utilized frequency ([21], [22], [23]). This helps minimize third-order intermodulation distortion, a common hardware effect in which interacting signals generate additional, unwanted frequencies that may interfere with communication.

Distributed Memory and High-Performance Computing. In multiprocessor systems and partitioned databases, avoiding communication conflicts is essential for efficient execution. Sum-free partitions, obtained by dividing a set of integers into sum-free subsets, can be used to organize parallel tasks and manage data distribution. Such partitions help prevent overlaps among dependent processes, thereby improving data integrity and reducing the likelihood of conflicts during computation.

Applications of MBF and SFS in Image Mining and Association Rule Mining. In data-mining applications, including image mining and association rule mining, sum-free systems can serve as mathematical constraints that reduce the search space and eliminate irrelevant patterns, or model pixel relationships.

In image processing, images are often transformed into vectoral embeddings (e.g., matrices of pixel intensities, color channels, or frequency coefficients). Within this context, sum-free properties may be utilized in:

- Feature Selection and Pruning: In the analysis of multispectral and hyperspectral images, pixel features are represented by combinations of frequencies. Sum-free conditions can be applied as structural rules to reduce redundant or overlapping harmonic features;
- Texture Analysis: In mathematical morphology and fractal-based image analysis, sum-free constructions may be used to design independent collections of structural descriptors. Such representations allow algorithms to analyze shape variations without producing overlapping morphological sums;
- Cryptography and Watermarking: Image mining often involves hiding data securely. Sum-free sets may provide a framework for constructing robust error-correcting codes ([3]) and watermarking keys, allowing copyright data to be embedded without causing collisions during the extraction;
- Association rule mining aims to discover dependencies and co-occurrence patterns by: Avoiding Pattern Collisions. An association rule is an implication of the form $X \implies Y$. To ensure rule stability and mathematical uniqueness, rules are often restricted to disjoint itemsets. The algebraic properties of sum-free sets ensure that the combined itemsets do not generate unresolvable or overlapping constraints;
- Constrained Candidate Generation: In large datasets with millions of items (e.g., market basket analysis), generating all possible itemsets requires intensive computation. Using sum-free properties limits the search space to only

subsets that satisfy the anti-monotonicity property, dramatically speeding up algorithms;

– Data Generalization: In quantitative association rule mining where items are continuous numbers (like prices or quantities), sum-free subsets are used as mathematical thresholds to group variables. This prevents item values from adding up to "collisions" (situations where the sum of antecedent components inappropriately defines a rule consequent).

Definitions. Let $[n] = \{1, 2, \cdots, n\}$. For $3 \leq k \leq n$, let $(i_1, i_2, \cdots, i_k)$ be a k-tuple of distinct elements of $[n]$. We say that $(i_1, i_2, \cdots, i_k)$ is *sum-free* if, for any element i_r of the tuple, there do not exist distinct indices $1 \leq j_1, j_2, \cdots, j_t \leq k$ in $\{1, \cdots, k\} \setminus r$, with $2 \leq t \leq k-1$ such that $i_r = i_{j_1} + i_{j_2} + \cdots + i_{j_t}$.

Equivalently, a tuple is sum-free if no element of the tuple can be expressed as the sum of distinct other elements of the tuple. A tuple that is not sum-free is called *non-sum-free*.

The sum-free property is inherited in the following sense: any subset of a sum-free set is also sum-free. Therefore, the family of sum-free sets can be described by monotone Boolean functions via the set of their zeros. Consequently, non-sum-free sets correspond to the set of units of these functions.

In this paper, we focus on the case $t = 2$.

It is clear, that these functions take the value 0 on all vertices of the zeroth first, and second layers. Therefore, the third layer is the lowest layer that may contain non-sum-free sets.

In this paper, we enumerate all (non-)sum-free sets on the third layer, that is, all (non-)sum-free triples of $[n]$. We also determine the largest k for which there exists a sum-free sets of size k in $[n]$.

The remainder of the paper is organized as follows. Section 2 introduces the necessary definitions. In Section 3, we determine the largest layer k that may contain zeros of the function corresponding to sum-free sets. In Section 4, we use a chain decomposition of $(P(n, 3), \preceq)$, where $P(n, 3)$ denotes the set of all increasing triples from $[n]$, and $\preceq$ is the componentwise partial order, to identify and enumerate all non-sum-free elements of $P(n, 3)$. As a consequence, we obtain the number of all the sum-free elements. Finally, Section 5 contains concluding remarks and outlines directions for future research on sum-free sets.

2 Preliminaries

This section introduces basic concepts of monotone Boolean functions, following standard references. We also present the necessary definitions and terminology from the theory of partially ordered sets (posets); see, for example, [9].

2.1 Monotone Boolean functions

Let $B^n = \{(x_1, \cdots, x_n) | x_i \in \{0, 1\}, i = 1, \cdots n\}$ denote the set of vertices of the n-dimensional binary cube. We define a componentwise partial order on B^n: for any $\alpha = (\alpha_1, \cdots, \alpha_n)$ and $\beta = (\beta_1, \cdots, \beta_n)$ of B^n, $\alpha \preceq \beta$, if and only if $\alpha_i \leq \beta_i$ for $1 \leq i \leq n$. Let $L_k = \{(\alpha_1, \cdots, \alpha_n) \in B^n \mid \sum_{i=1}^{n} \alpha_i = k\}$. We call L_k the k-th layer of B^n.

A Boolean function $f : B^n \to \{0, 1\}$ is *monotone* if, for every pair of vertices $\alpha, \beta \in B^n$, $\alpha \preceq \beta$ implies $f(\alpha) \leq f(\beta)$. The vertices of B^n where f takes the value "1" are called units of the function; vertices, where f takes the value "0" are called zeros.

α^1 is called a lower unit of the function if $f(\alpha^1) = 1$, and $f(\alpha) = 0$ for every $\alpha \in B^n$, such that $\alpha \prec \alpha^1$.

α^0 is called an upper zero of the function if $f(\alpha^0) = 0$, and $f(\alpha) = 1$ for every $\alpha \in B^n$ such that $\alpha^0 \prec \alpha$.

2.2 Partially Ordered Sets

A partially ordered set (or poset) is an ordered pair $(P, \preceq)$, where P is a set and $\preceq$ is a binary relation on P satisfying the following properties: reflexivity, antisymmetry, and transitivity. We write $x \prec y$ when $x \preceq y$ and $x \neq y$.

An element x of P is called minimal if there is no element $y \in P$ such that $y \prec x$. Similarly,x is maximal if there is no element $z \in P$ such that $x \prec z$.

Two elements $x, y \in P$ are comparable if $x \preceq y$ or $y \preceq x$; otherwise, they are incomparable. A *chain* in a poset $(P, \preceq)$ is a subset of P in which every pair of elements is comparable. An *antichain* is a subset of P in which no two elements are comparable.

The height of a poset is the cardinality of its largest chain; the width is the cardinality of its largest antichain. We denote the height and width of a poset $(P, \preceq)$ by $h(P)$ and $w(P)$, respectively. In a finite poset, a chain and an antichain can share at most one common element.

Theorem 1. *(Dilworth's Theorem, [9]) Let $(P, \preceq)$ be a finite poset. Then there is a partition of P into $w(P)$ chains.*

Let x and y be distinct elements of P. We say that y covers x if $x \prec y$ but no element z such that $x \prec z \prec y$. The Hasse diagram of a poset $(P, \preceq)$ is a directed graph whose vertex set is P, with edges corresponding to covering pairs. In practice, the Hasse diagram is typically drawn so that if y covers x, the point representing y is placed higher than the point representing x. In this representation, the direction of each edge is implicit, and arrows are usually omitted.

The poset that we investigate in this work, is a special order of k-element subsets of a finite set. This structure corresponds to the k-th layer of the Boolean cube.

Let $P(n, k)$ denote the set of all k-tuples with strictly increasing elements from the set $[n] = \{1, 2, \cdots, n\}, 1 \leq k \leq n$:

$$P(n, k) = \{(i_1, \cdots, i_k) | 1 \leq i_1 < \cdots < i_k \leq n\}.$$

The number of elements is $|P(n,k)| = \binom{n}{k}$.

We define a component-wise partial order on $P(n,k)$:
for two elements $(i_1, \cdots, i_k)$ and $(j_1, \cdots, j_k)$ of $P(n,k)$, $(i_1, \cdots, i_k) \preceq (j_1, \cdots, j_k)$ if and only if $i_1 \leq j_1, \cdots, i_k \leq j_k$. Thus, $(P(n,k), \preceq)$ is a poset, whose minimal element is $(1, 2, \cdots, k)$, and maximal element is $(n-k+1, \cdots, n)$. We define the weight of $(i_1, \cdots, i_k)$ as the sum of its coordinates, $i_1 + \cdots + i_k$.

To each $(a_1, \cdots, a_k) \in P(n,k)$, we assign a binary sequence $(\beta_1, \cdots, \beta_n)$, where $\beta_i = 1$ if and only if $i \in \{a_1, \cdots, a_k\}$. This mapping yields the set of vertices in the k-th layer of the binary cube B^n. For example, the element $(2,3,5)$ of $P(5,3)$ corresponds to the vertex (01101) in B^5.

3 Largest sum-free sets in $[n]$

In this section, we determine the largest k for which there exists a sum-free sets of size k in $[n]$. Equivalently, we identify the highest layer of the hypercube B^n that contains a vertex corresponding to a sum-free k-element subset of $[n]$. To this end, we examine the configurations that give rise to non-sum-free sets.

Without loss of generality we consider k-tuples with strictly increasing elements from $[n]$. Recall that we restrict attention to the following (2-element) case of sum-free tuples:

a k-tuple $(i_1, i_2, \cdots, i_k)$ with $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ is sum-free if, for every $j \leq k$ there do not exist indices $1 \leq j_1 < j_2 < j$ such that $i_j = i_{j_1} + i_{j_2}$.

3.1 Relation with Monotone Boolean functions

The sum-free property is inherited in the following sense: any subset of a sum-free set is also sum-free. Therefore, the family of sum-free sets can be described by monotone Boolean functions via the set of their zeros. Consequently, non-sum-free sets correspond to the set of units of these functions. The lower units of the function correspond to non-sum-free subsets of minimum size.

Suppose, that $(i_1, i_2, \cdots, i_k)$ is a sum-free k-tuple. We associate with it the vertex of B^n in the k-th layer whose coordinates are 1 in positions $i_1, i_2, \cdots, i_k$, and 0 elsewhere.

We now consider all possible values of i_k, equivalently, all possible positions of the last 1 in the corresponding vertex.

Clearly, $i_k \geq 3$. Therefore, these functions take the value 0 on all vertices of the zeroth, first, and second layers. Consequently, the third layer is the lowest layer that can contain vertices corresponding to non-sum-free sets. The highest layer is the n-th layer.

If $i_k = 3$, the only possible non-sum-free tuple is $(1,2,3)$; if $i_k = 4$, it is $(1,3,4)$, etc., if $i_k = 7$, the possibilities are $(1,6,7)$, $(2,5,7)$, and $(3,4,7)$, and so on.

Table 1 lists the integers together with their representations as sums of distinct smaller integers. The left column contains the integers,while the right column gives their representations as sums of distinct terms written in increasing order.

Table 1. Representations of integers as sums of increasing terms

Integer	**Representations as sums of increasing terms**
3	$1+2$
4	$1+3$
5	$1+4,\ 2+3$
6	$1+5,\ 2+4$
7	$1+6,\ 2+5,\ 3+4$
$\vdots$	$\vdots$
n	$\begin{cases} 1+(n-1),\ 2+(n-2),\ 3+(n-3),\ \ldots,\ \left(\frac{n}{2}-1\right)+\left(\frac{n}{2}+1\right), & \text{if } n \text{ is even,} \\ 1+(n-1),\ 2+(n-2),\ 3+(n-3),\ \ldots,\ \left(\frac{n-1}{2}\right)+\left(\frac{n+1}{2}\right), & \text{if } n \text{ is odd.} \end{cases}$

The largest number of terms is obtained for n, meanwhile, this is $\frac{n}{2}-1$ for even n, and $\frac{n-1}{2}$, for odd n.

To avoid sums, we must exclude at least one from each such pair.

Returning to the layers of the hypercube, a vertex corresponds to a sum-free set only if it contains at least $\frac{n}{2}-1$ or $\frac{n-1}{2}$ zeros, depending on the parity of n, that is, $\lceil \frac{n}{2} \rceil - 1$ zeros. Therefore, such a vertex can contain at most $n - (\lceil \frac{n}{2} \rceil - 1)$ ones. This determines the highest layer that may contain sum-free subsets.

Theorem 2. *The highest layer of B^n that contains vertices corresponding to sum-free-sets is $n - \lceil \frac{n}{2} \rceil + 1$.*

As an example, let $n = 5$ and consider sum-free subsets in B^5.

Vertices (10011), (01101), (10110), and (11100) in the third layer are not sum-free; the remaining vertices (00111), (01011), (01110), (10101), (11001), and (11010) are sum-free. According to Theorem 2, all vertices in higher layers, are not sum-free. On the other hand, all vertices of the zeroth, first, and second layers are sum-free.

4 Sum-Free and Non-Sum-Free Triples

Since the third layer is the lowest layer that may contain non-sum-free sets, in this section we begin by characterizing all sum-free and non-sum-free triples of $[n]$. Equivalently, we characterize all sum-free and non-sum-free vertices in the third layer of B^n. For this reason, we focus on $P(n, 3)$.

4.1 $P(n, 3)$ and Chain Decomposition

$P(n, 3) = \{(i_1, i_2, i_3) : 1 \leq i_1 < i_2 < i_3 \leq n\}$ is the set of all triples of increasing elements from $[n]$.

The Hasse diagram of $P(n, 3)$ is naturally layered: it has $3(n-3)+1$ layers, indexed from 0 to $3(n-3)$,

- The lowest layer consists of the unique minimal element $(1, 2, 3)$,
- Each successive layer l consists of all elements that cover some element in layer $l-1$.

– The highest layer contains the unique maximal element $(n-2, n-1, n)$.

Within each layer, elements are ordered lexicographically, with the smallest element positioned on the right. Each element in layer l has weight $l+(1+2+3)$.

– If $3(n-3)$ is even, there is a unique middle layer, namely the $(3(n-3))/2$ -th layer.
– If $3(n-3)$ is odd, there are two middle layers, namely the $(3(n-3)-1)/2$ -th and $(3(n-3)+1)/2$-th layers.

In [17] it was shown that the width $w(P(n,3))$ is achieved by the largest layer; and an explicit construction of the covering chains is also given.

The chains for the example $(P(9,3), \preceq)$ are as follows.

Chain 1: 123,124,125,126,127,128,129,139,149,159,169,179,189,289,389,489, 589,689,789
Chain 2: 134,135,136,137,138,148,158,168,178,278,378,478,578,678
Chain 3: 234,235,236,237,238,239,249,259,269,279,379,479,579,679
Chain 4: 145,146,147,157,167,267,367,467,567
Chain 5: 245,246,247,248,258,268,368,468,568
Chain 6: 156,256,257,357,358,458,459
Chain 7: 345,346,347,348,349,359,369,469,569
Chain 8: 356,456,457.

Our goal is to identify non sum-free triples using $(P(n,3), \preceq)$ and the chains, as well as their positions on the chains. For example, in the Chain 1, we have two non-sum-free triples, these are (1,2,3) and (1,8,9); in the Chain 2, these are (1,3,4) and (1,7,8), etc.

4.2 Decomposition of $P(n,3)$ via Pascal's Identity

First, we partition $P(n,3)$ into the sets $P(j,2)$, $j = 2, \cdots, n-1$, where $P(j,2)$ denotes the set of all 2-element subsets of $[j] = \{1, \cdots, j\}$.

$P(j,2) = \{(i_1, i_2) : 1 \leq i_1 < i_2 \leq j\}$, and $|P(j,2)| = \binom{j}{2}$.

We apply the following Pascal's identity:

$$\binom{n}{r} = \binom{n}{r-1} + \binom{n-1}{r-1}.$$

iteratively to decompose $P(n,3)$ into subsets $P(n-1,2)$, $P(n-2,2)$, $P(n-3,2)$, $\cdots$, $P(3,2)$, $P(2,2)$.

For $P(n,3)$ we get:

$$\binom{n}{3} = \binom{n-1}{2} + \binom{n-2}{2} + \binom{n-3}{2} + \cdots + \binom{3}{2} + \binom{2}{2}.$$

For example, $P(9,3)$ will be partitioned in the following way:

$P(8,2) = \{123, 124, 125, 126, 127, 128, 129,$
$134, 135, 136, 137, 138, 139,$

$145, 146, 147, 148, 149,$
$156, 157, 158, 159,$
$167, 168, 169,$
$178, 179,$
$189\}$
$P(7, 2) = \{234, 235, 236, 237, 238, 239,$
$245, 246, 247, 248, 249,$
$256, 257, 258, 259,$
$267, 268, 269,$
$278, 279, \ 289\}$
$\ldots$
$P(3, 2) = \{678, 679, 689\}$
$P(2, 2) = 789$
A geometrical illustration is given in Figure 1.

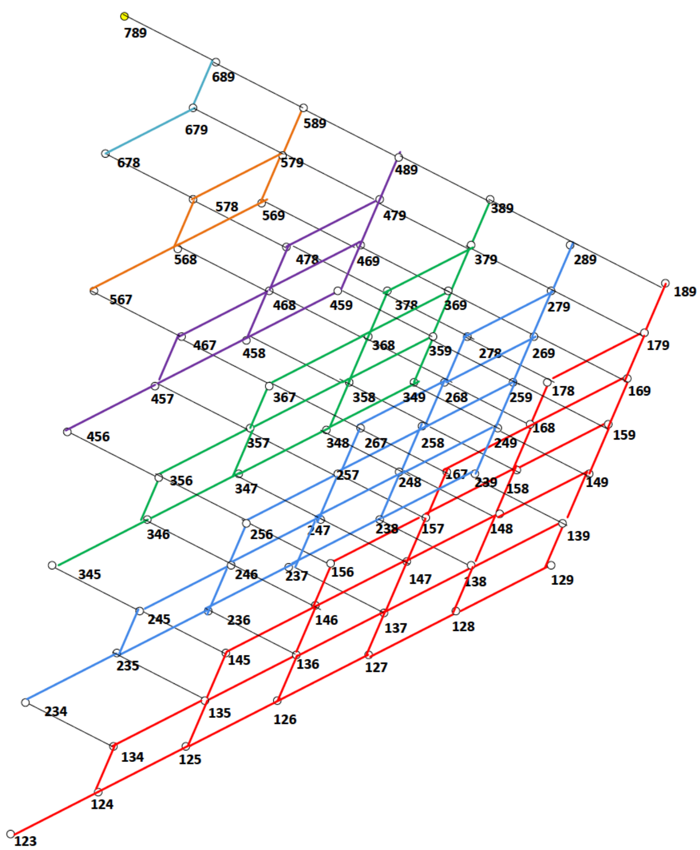

Figure 1. Partitioning of $P(9, 3)$.

4.3 Identifying Non-Sum-Free Elements via Further Decomposition

Each $P(j, 2)$ can be partitioned into disjoint chains.

For $P(n-1,2)$:
There are $n-2$ subchains, and the first element of each chain is not sum-free.
Subchain 1: $\{123, 124, 125, 126, 127, 128, \cdots, 12n\}$ (the length is $n-2$)
Subchain 2: $\{134, 135, 136, 137, 138, \cdots, 13n\}$ (the length is $n-3$)
...
Subchain $n-2$: $\{1\ n-1\ n\}$ (the length is 1).
Hence, there are $n-2$ non sum-free elements for $P(n-1,2)$.

For $P(n-2,2)$:
There are $n-3$ subchains, and the second element of each chain is not sum-free.
Subchain 1: $\{234, 235, 236, 237, 238, \cdots, 23n\}$
Subchain 2: $\{234, 235, 236, 237, 238, \cdots, 23n\}$,
Subchain 3: $\{245, 246, 247, 248, \cdots, 24n\}$,
...
Hence, there are $n-4$ non sum-free elements in $P(n-2,2)$.

In general, for $P(n-j,2)$:
The number of subchains is $n-(j+1)$, and the j-th element of each chain is not sum-free.
Thus, the number of such elements is $n-2j$ in $P(n-j,2)$, for as long as $n-2j>0$.

4.4 Counting Non Sum-Free Elements

We sum $(n-2)+(n-4)+(n-6)+\cdots+(n-2j)$,
where j is the largest integer such that $j < \frac{n}{2}$.
Hence,

$$\sum_{k=1}^{j}(n-2k) = jn - 2\frac{j(j+1)}{2} = jn - j(j+1).$$

Since $j = \frac{n}{2} - 1$ when n is even, and $j = \frac{n-1}{2}$ when n is odd, we obtain the following result.

Theorem 3. *The number of triples $(i_1, i_2, i_3) \in P(n,3)$ satisfying $i_3 = i_1 + i_2$ is*

$$\begin{cases} \frac{n}{2}\left(\frac{n}{2}-1\right), & \text{if } n \text{ is even,} \\ \left(\frac{n-1}{2}\right)^2, & \text{if } n \text{ is odd.} \end{cases}$$

As Theorem 3 states, the number of non-sum-free triples is either $\frac{n}{2}\left(\frac{n}{2}-1\right)$ or $\left(\frac{n-1}{2}\right)^2$, depending on the parity of n, and these triples have been explicitly determined. Consequently, all other points in the third layer are sum-free.

5 Conclusion

In this paper, we investigated sum-free subsets of $[n]$ through the framework of monotone Boolean functions. Exploiting the hereditary nature of sum-free sets, we established a natural correspondence between the family of sum-free sets and the zero set of a monotone Boolean function. Focusing on the classical notion of sum-freeness, where no element is the sum of two others, we analyzed the structure of the associated vertices in the Boolean cube.

Our main results determine the highest layer of the Boolean cube B^n that contains vertices corresponding to sum-free sets, showing that this layer is $\lfloor n/2 \rfloor + 1$. In addition, we characterized and enumerated all non-sum-free triples of $[n]$. Using this characterization, we derived the number of sum-free triples and obtained a clearer understanding of the distribution of sum-free configurations within the Boolean cube.

These findings reveal a direct structural connection between extremal properties of sum-free sets and the theory of monotone Boolean functions. Beyond their theoretical interest, such connections may prove useful in applications involving frequency assignment in wireless communication, distributed computing, discrete tomography and image analysis ([13], [14], [15]), and association rule mining, where sum-free structures naturally arise as mechanisms for avoiding conflicts, redundancies, and undesirable interactions.

Future research may extend these methods to the general case in which an element is forbidden from being represented as the sum of t other elements, $t \geq 3$, and investigate the corresponding monotone Boolean functions and their combinatorial properties.

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