Automating the Construction of Formalized Representations of Initial Information in Descriptive Image Analysis

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Abstract. This paper continues the development of the Descriptive Theory of Image Analysis proposed by the authors and is devoted to automating the construction of formalized representations of initial information in descriptive image analysis. Automating the extraction of information from images requires a comprehensive use of all the capabilities of the mathematical apparatus in the tasks of image processing, analysis, recognition, and understanding. The authors of the paper develop descriptive image analysis - a set of descriptive methods and models designed to analyze and evaluate information presented in the form of images. The concepts of image realizations/representations/models, descriptive algorithmic schemes for image representation, models that characterize the architecture and structure of the image analysis and recognition process – a multi-level model for combining algorithms and initial data, and an information structure for generating descriptive algorithmic schemes are considered. A new mathematical object is introduced – the image formalization space - to formalize and standardize the description of all acceptable states and procedures for the process of reducing images to a form that is convenient for recognition, in the form of a single model corresponding to a certain mathematical space. The main methodological principles of automating the construction of formalized representations of initial information in descriptive image analysis are presented.

Keywords: Image Analysis, Descriptive Image Analysis, Mathematical Theory of Image Analysis, Automation of Information Extraction from Images, Image Recognition, Descriptive Algorithmic Schemes, Models of Image Analysis Procedures, Image Representations, Image Models, Image Recognition Algorithms, Mathematical Structures, Multiple Classifiers, Dual Representations of Images

1 Introduction

This research continues the development of the Descriptive Theory of Image Analysis proposed by the authors and is devoted to automating the construction of formalized representations of source information in descriptive image analysis (DA) [1-14]. Due to close attention to the study of mathematical properties of the image formalization space (IFS) [12, 13], this paper outlines the main results obtained by the authors in the field of formalization of initial and transformed data in the process of image analysis (IA).

Automating the construction of initial data representations in IA is a critical area of scientific research. It resolves the fundamental contradiction between the complexity of the visual world and the need for its formalization for machine analysis.

In our understanding, automating the extraction of information from images requires a comprehensive use of all the capabilities of the mathematical apparatus in the problems of image processing, analysis, recognition, and understanding. Therefore, our research focuses on the theoretical aspects of IA.

DA is a logically organized set of descriptive methods and models designed to analyze and evaluate information presented in the form of images, and to automate the extraction of knowledge and data from images necessary for making intelligent decisions about real-world scenes displayed and represented using the analyzed image.

The first section of the paper is devoted to some aspects of existing theoretical approaches in IA.

The second section of the paper is devoted to the image as an object of analysis, and formalized representations that are built in the framework of solving IA problems. Image representations/models are considered within the framework of the proposed DA.

The third section of the paper is devoted to the processes of IA in DA. One of the key objects of DA is descriptive algorithmic schemes (DAS) [4, 7, 9], in particular, DAS for Image Representation (DASIR) [4]. To automate the construction of DAS in DA, two models are proposed that characterize the architecture and structure of the image analysis and recognition process – a multi-level model for combining algorithms and initial data (MMCAI) [8] and information structure of generation (ICG) DAS [9].

The fourth section is devoted to a new mathematical object defined in the DA – the image formalization space (IFS). IFS was introduced in [4] in order to formalize and standardize the description of all acceptable states and procedures for image reducing to a form convenient for recognition (IRRF) in the form of a single model corresponding to a certain mathematical space. In connection with the definition and use of IFS, there are problems related to the study of IFS as a mathematical space [12, 13].

Figure 1 shows the main methodological principles for automating the construction of formalized representations of initial information in DA:

1) principle 1: "algebraization" of image analysis, which allows generating new ones from acceptable image transformations by applying specified algebraic operations (DA uses the mathematical language of descriptive image algebras (DIA) [6, 11] to represent DASIR);

2) principle 2: the use of image representations using the so-called "combinatorial structure of local neighborhoods "(CSLN representations), which takes into account the two-dimensional nature of images and allows you to study image fragments without identifying features on them;

3) principle 3: the use of image models, which may not always be used to restore the original image, but at the same time it is possible to successfully solve the IA problem (for this formalization, descriptive image models (DIM) are selected in DA);

4) principle 4: generating new multi-model and multidimensional representations of images by combining several DIM, or parts of them;

5) principle 5: when constructing formalized representations of images, the use of MMCAI based on the joint use of methods for combining algorithms and methods for combining fragmentary initial data – partial descriptions of the object of analysis and recognition-images; the use of dual representations of images (CLSN-representations/models of images);

6) principle 6: ICG DAS is a means of representing and implementing information processing processes when solving the problem of image recognition in any formulation and arbitrary scenarios, models and methods of solution and provides emulation of any DAS and their combinations used and generated in the process of solving the problem of image recognition;

7) principle 7: DAS can: a) be defined by the selected scenario for solving the image recognition problem; b) be formed by defining and combining information processing procedures when modeling the processes of solving the recognition problem on the ICG DAS;

8) principle 8: The IFS is considered as a logically conceivable structure that serves as the environment in which the IRRF is implemented.

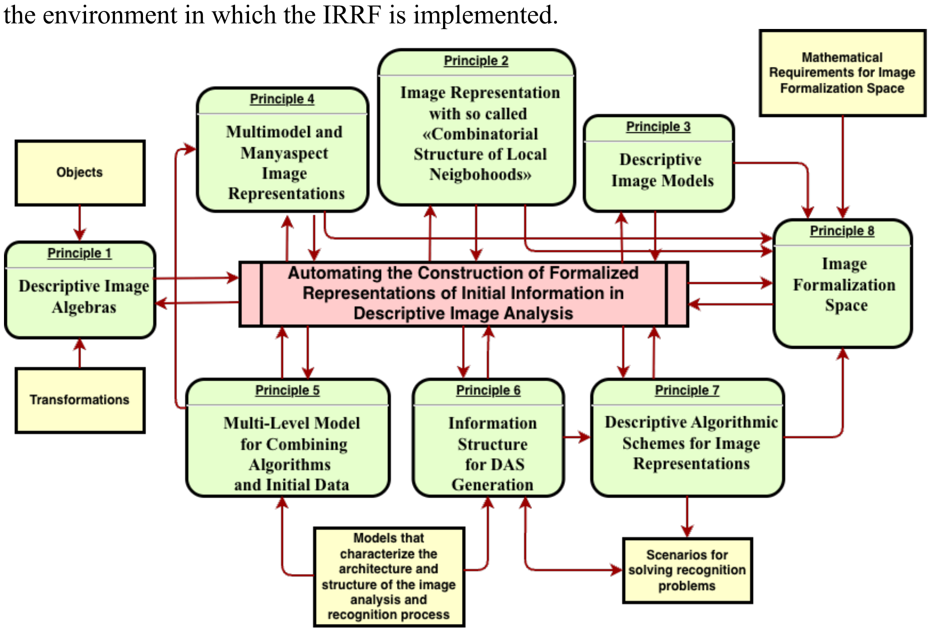

Fig. 1. Methodological principles for automating the construction of formalized representations of initial information in descriptive image analysis

2 Current Trends in Theoretical Image Analysis

Below is a brief overview of the key theoretical directions and research results in the field of raw data representation in image analysis in the world. The review does not claim to be complete, but only some aspects of image formalization are considered.

The main results and the overall contribution of Russian scientific schools to the current global level of development of mathematical theory and applications of image analysis and pattern recognition are published in [15, 16].

The first area of research on automating the formation of image representations is related to the redundancy of initial data in image analysis, if the latter is considered as a set of points. To apply machine learning algorithms, it is necessary to find such representations of images that, with a smaller dimension, preserve significant structures.

For these purposes, for example, the principal component method (PCA) was introduced as a theoretically valid method for selecting a subspace with maximum variance [17]. Other ways to reduce the dimension of data are the Independent Component Analysis method [18], multidimensional scaling [19], as well as numerous nonlinear generalizations: the method of principal curves and manifolds, Projection Pursuit, neural network methods of "narrow throat", self-organizing Kohonen maps, etc.

Another way to reduce the image dimension is the Image Manifold concept [20], which assumes that all possible images of an object (for example, faces in different lighting conditions, rotations) lie on a low-dimensional smooth manifold embedded in a high-dimensional space. Research is focused on methods of its approximation (ISOMAP, LLE, t-SNE, UMAP) and the study of its metric properties.

The next area of research on various representations of images is related to image properties that should be ignored, or vice versa preserved. This resulted in the theory of invariants:

1. Moment invariants (Hu, Zernike, Legendre) [21]: a strict algebraic theory for constructing features insensitive to groups of geometric transformations (shift, rotation, scale). The questions of completeness and minimality of the obtained basis of invariants are investigated.
2. Convolutional neural networks (CNNs) [22] as an architectural invariant: the theoretical justification for the success of CNNs is based on the introduction of a priori knowledge of equivariance to shift (convolution) and invariance to local distortions (pooling). Equivariance is a property of functions or systems in which their behavior is preserved under certain symmetry transformations. CNN-implementation of group representation theory in the network architecture.
3. Theory of scattering transformations [23]: construction of deep invariant representations based on wavelet transforms and nonlinear modulus operators. The Lipschitz continuity of such representations with respect to small diffeomorphisms (deformations) is theoretically proved.

The next direction of development in the study of image representation formation is syntactic (structural) and logical representations.

An alternative to vector representations is to describe an image as a structure consisting of non-derived elements and relationships between them:

1. Generative image grammars: the theoretical basis goes back to Chomsky and Fu. The image is not classified, but "parsed" like a sentence in a language. Formalisms

include graph grammars, pool grammars, and stochastic form grammars. The main theoretical problem is Bayesian inference of optimal grammar and scene structure (parsing) from pixel data.

2. Relational models and graphs: a formal representation in the form of an attribute graph, where vertices are segments and edges are their spatial relationships ("above", "inside", "touches"). The theory explores algebraic methods for matching such graphs (for example, through spectral graph theory or random walks), which is a formal basis for solving the recognition problem.

Another area that has a mathematical basis and neurophysiological physical meaning is sparse representation theory (Sparse Coding) [22]. Search for such a decomposition of the signal (image) using a redundant dictionary (basis), which gives the most sparse vector of coefficients.

The current trend of using neural network technologies to solve various classes of image analysis problems has led to a shift in research from methods of constructing image representations to studying the properties of trained objects.

The following approaches are known:

1. Information-theoretical approach (Information Bottleneck) [25]: the image is considered as a noisy channel, and the task of the network is to sequentially compress the input data, discarding irrelevant information (noise, variations) and preserving relevant information (object class). The theory analyzes the dynamics of mutual information between layers, inputs and outputs.
2. Disentangled Representations [26]: one of the key theoretical goals of deep generative models (β-VAE, FactorVAE). Building such a formal representation (latent code), where each independent dimension is responsible for one meaningful factor of data variation (for example, separately for the shape, separately for lighting, separately for the angle). Metrics and theoretical limits of unsupervised unraveling are investigated.

Category – theoretic approach and functional programming are the least developed but growing areas that strive for extreme generality.

An image and its representations are treated as objects and morphisms in a category. Automation of view construction is the search for a mapping that translates pixel data into" semantic " categories while preserving the structure. This approach offers a single language for composing heterogeneous representations (probabilistic, geometric, and structural), which is important, for example, in robotics.

Global trends in the field of theoretical IA have gone from the engineering design of features (moments, Gabor filters) to the search for optimal mathematical structures (manifolds, sparse decompositions, invariant groups) and, as a result, to the study of the properties of trainable representations in deep neural networks. The key modern theoretical question is not "how to manually build a good representation", but "what are the mathematical principles that guarantee that a trained model will build a representation that is generalizable, robust and interpretable". Heuristic methods are increasingly returning researchers to their roots-mathematical foundations.

3 Initial Data Representation

An image is inherently an object with a complex information structure that reproduces information about the original scene using the brightness values of discrete image elements - pixels, configurations of image fragments, sets of pixels, and spatial and logical relationships between configurations, sets of pixels, and individual pixels. An image differs from other means of data representation by its high information content, clarity, structure, and natural perception of its content for a person.

The image is a mixture of the original (raw, "real") data, their realizations, and some deformations. Implementations (as well as the corresponding descriptions) reflect the informational and physical nature of objects, events, and processes reproduced using an image, and deformations are generated by the technical characteristics of the means by which the image is registered, formed, and transformed in the process of constructing digital images and the hierarchy of their descriptions.

The initial information presented in the form of images is poorly structured and unformalized. A common method of standardization in applied mathematics and computer science is the construction and use of mathematical and simulation models of the studied objects and procedures used for their transformation.

When defining the system of concepts of DA, the following properties of images were fundamental:

1. the image consists of a set of points and a set of values associated with these points.
2. transformation of images for the purpose of their analysis includes operations on images and on various values and characteristics associated with these images;
3. the image is characterized by two types of information: it is defined by the spatial relationships between points and certain types of numeric or other descriptive information associated with these points.
4. a set of points is a topological space (it consists of objects called points and a topology that defines the proximity of two points, the connectivity of a subset of the set of points, the neighborhood of points, the presence of boundary points, curves and arcs).

When formalizing an image as a recognition object ,the" content " of the image is characterized by the following types of information:: a) identifiable objects with a well-defined structure; b) identifiable objects with a poorly defined structure; c) unidentifiable objects; d) identifiable areas of interest with a well-defined structure; e) identifiable areas of interest with a poorly defined structure; f) background, i.e., in general, everything else.

In DA, there are two fundamentally different types of image representation:

1. using the system of objects and relations selected in the image [3, 7] (DIM);
2. using CSLN representations - the actual spatial information (for example, fragments of an image - pixels or sets of pixels that form a local neighborhood of some order in the image) [8].

Semantic information presented in the image or associated with the reproduced scene allows you to use a variety of contextual information for analysis and recognition (forbidden and impossible order relations, partial order relations, and other conditions and restrictions that reflect the physical and logical organization of the real world). To apply the recognition algorithm, you must use the IRRF procedures.

In the first case, the IRRF procedures should provide the construction of a mathematical model of the image. In DA, DIM-parametric, procedural, and generative models-are selected as formalized models.

In the second case, the recognition algorithm must accept the source data that represents the image itself or its fragments [27]. For recognition, "spatial" algorithms are used (algorithms that allow spatial information as the source), and the IRRF procedures are generally reduced to image segmentation, choosing the shape of fragments that are used to compare the recognized and reference objects, splitting the image into fragments, and so on.

The recognition algorithms used to process each of the two types of image representation differ.

An image model can include structural or at least relational information. In this case, it is a formal construction that consistently takes into account the hierarchical structure of the recognition object and the relationships that exist between individual elements of this hierarchy, both within the same levels and between them. The DA introduces a structure designed for classifying and automating the generation of image models – generative descriptive trees.

By combining descriptors from one or more generative descriptor trees, mixed-type image models are constructed that reflect the properties of the original image in various aspects. The introduction of generative descriptive trees makes it possible to generate multi-model representations of images and, consequently, combine, merge, and combine multidimensional image characteristics in the recognition process.

4 Descriptive Algorithmic Schemes for Image Recognition

DA provides the ability to solve both problems related to constructing formal descriptions of images as recognition objects, and problems of synthesizing image recognition and understanding procedures. The operational approach to image characterization provides that, in general, the processes of analyzing and evaluating information presented in the form of images – problem solving trajectories-can be considered as a certain sequence/combination of transformations and calculating a certain set of intermediate and final (decision-making) estimates.

The set of valid algorithmic image transformation schemes forms a set of DAS.

DAS [9] is a certain transformation system applied to the source image that implements mathematical methods for image processing, analysis, and recognition and provides:

a) construction of a model or representation of an image that allows the use of classical or" spatial "recognition algorithms, by means of sequential transformation of the" states "of the processed image corresponding to the degree of its current "formalization";

b) solving the problem of image recognition by applying recognition algorithms to the image model/representation constructed in accordance with paragraph " a " of this definition.

Models of image recognition and analysis processes reflect the problem statement, the mathematical and heuristic methods used, and the algorithmic content of the process. These models include: a model based on reverse algebraic closure [2]; a model based on the equivalence property of images of the same class [21]; a model based on multiple image models and multi-algorithmic classifiers (MAC) [3].

The model of image recognition and analysis based on reverse algebraic closure provides for a multi-level and multidimensional structure of the recognition process. All transformations are implemented by the method of reverse algebraic closure.

The image equivalence model is based on the following assumptions:

- the image is a partial representation of some physical reality (scene);
- on a set of images, you can define an equivalence relation that corresponds to the image belonging to the same scene. This model leads to 2 new formulations of the image recognition problem.

A model based on multiple models and algorithms implements the principles of multialgorithmicity and multimodelism and uses the merging of results obtained for individual algorithms and models. The model provides for:

- parallel application of several different algorithms for processing the same model and several different models of the same source data;
- combining the results to increase the exact solution.

Models that characterize the architecture and structure of the speech recognition process include:

- MMCAI [8, 14];
- ISG DAS [9, 14].

The main properties of the MMCAI class are: a) combining fragments of initial data and their representations and combining algorithms at all levels of image analysis and recognition processes; b) using multi-algorithm schemes in the process of image analysis and recognition; c) using dual representations of images as input data for analysis and recognition algorithms.

MMCAI is introduced in order to provide the possibility of: a) standardization, modeling and optimization of the DAS that form the algorithmic content of the MMCAI, and process heterogeneous poorly formalized information-dual representations-spatial, symbolic and numerical representations of the initial data; b) comparative analysis, standardization, modeling and optimization of various algorithms for analyzing and recognizing spatial information.

The introduction of MMCAI as a standard structure for the representation of algorithms for analyzing and recognizing 2-dimensional information and dual representations of images allows us to generalize and justify well-known heuristic recognition algorithms and evaluate their mathematical properties and applied utility.

Key features of the MMCAI architecture:

- joint use of MAC [28-30] and dual representations of source data;
- combining data and algorithms at 3 levels of processing, analysis, and recognition of source data, including images.

To regularize the generation of DAS for image analysis and recognition, a ISG DAS is introduced that has the following functionality: 1) solving the problem of image recognition in a given formulation, with given initial data, and using a given scenario that determines the sequence of application of information processing procedures and their iterative cycles; 2) constructing a DAS for solving the problem with given initial data in the absence of a given scenario; 3) in case of successful solution of the problem, fixing the sequence of procedures and cycles 4) comparative analysis and optimization of methods for solving image recognition problems by implementing them in the form of DAS and scenarios allowed by the structure.

5 Image Formalization Space

The IFS is considered as a logically conceivable structure that serves as an environment in which the IRRF is implemented.

Image analysis processes in DA are considered as sequences of transformations taken from and implemented in a certain space and providing the construction of states of image representations that together form the trajectory of image translation from the original view to the model. Recall that the image representation state is the current form of image representation, which is an element of the path of transformation of the original image into a descriptive image model.

DASIR is a formal scheme designed to provide a standardized formal description of surfaces, point configurations, shapes that form an image, and relations between them. Recall that the state of the image representation transformation model is the current DASIR form, which is an element of the full DASIR.

States of image representation transformation models can be DASIR with a different number of blocks containing image transformations; comparison blocks for evaluating the performance of the entire image processing, analysis, and recognition cycle; and loops that improve the results of applying DASIR to image representations and models.

DA provides for the implementation of image analysis processes in the IFS, the elements of which are various forms (states, phases) of the image representation that is transformed from the original view to a view that is convenient for recognition (i.e., into a model), and various forms (states, phases) of image representation transformation models.

In [13], the composition of IFS is considered in more detail. The hierarchical structure of the IFS organization is presented. Basic objects and relationships between them are described.

The IFS at the top level of the hierarchy is divided into two subspaces: the state space of image representations, and the state space of image representation transformation models.

The points of the state space of image representations are the "initial image", "image realizations", “intermediate states "of the image”, “parametric image model", "procedural image model", "generating image model", representation of the image using the so-called "combinatorial structure of local neighborhoods", and multi-model representation of the image. A detailed description of the process of constructing an image model and defining the concepts used above is presented in [3-7].

The points of the state space of image representation transformation models are "procedural transformations", "parametric transformations", "generative transformations"," procedural DAISR"," parametric DASIR"," generative DASIR", “intermediate states" of DASIR, and complete DASIR models.

The" geometry " of a IFS is defined by the relationships between subspaces described above and between different states of image representations/transformation models of image representations within subspaces.

At the second level of the hierarchy, the state space of image representations is divided into images, image representations, and image models.

At the second level of the hierarchy, the state space of image representation transformation models is divided into valid image transformations and DASIR.

At the third level of the IFS hierarchy, image representations are divided into the blocks "Intermediate states of image representation", "Representations of the form" combinatorial structures of local neighborhoods" and "Multi-model representations of images". There are three classes of image models: procedural, parametric, and generative. DASIR can also be of three classes: parametric, generative, and procedural; but they also take into account intermediate states of DASIR; and full DASIR. Acceptable image transformations can also be of three classes: procedural, parametric, and generative.

Above, we have introduced and defined the basic objects, the presence of which in the IFS is a necessary condition for the existence of a mathematical space that provides the definition and implementation of the processes of IRRF, that is, the process of constructing descriptive image models. These objects must be IFS points. As relations between points and collections of IFS points, we will consider logical operations of union/intersection/following and algebraic operations of addition/product/multiplication by a field element, that is, operations used in constructing data transformation algorithms. Based on this informative description of the purpose and function of the IFS, we define it as follows.

Definition [13]. IFS is a mathematical space that includes subspaces of states of image representations and states of transformation models of image representations. These states are points of subspaces of the IFS. Relations between points are defined by standard algebraic and logical data transformation operations in the process of constructing their formalized descriptions.

In the publication [12], the problems that arise in connection with the construction of a mathematical model of IFS in a standard mathematical space (metric, topological, phase) are defined, their statements are given, and the necessary conditions that the corresponding IFS models must satisfy are considered.

The priority tasks of studying the mathematical properties of IFS are highlighted:

1. Determination of the possibility of representing IFS as a set of subspace hierarchies with metric properties.
2. Determination of the possibility of representing IFS at the first (highest) level of the hierarchy in the form of two disjoint subsets: a) images and their representations, b) transformation schemes.

3. Determining whether the subsets described above can be divided into subsets at the second level of the hierarchy.
4. Determine the necessary conditions that must be met by the metrics entered at each of these subsets for individual levels of the hierarchy.
5. Checking whether standard metrics, such as Euclidean, Levenshtein, Hausdorff, and others used in image analysis and pattern recognition, can be used as metrics when constructing a IFS model in the metric space.
6. Checking the permissibility of using isometries in the IFS and/or in its subspaces, and/or between subspaces.
7. Checking the permissibility of using homeomorphisms in the IFS and/or in its subspaces and/or between subspaces.
8. Verification of the permissibility of using additional algebraic structures in the IFS (for example, DIA).
9. Determination of the possibility of representing IFS in the form of a set of subspace hierarchies with topological properties.
10. Determination of the possibility of representing each subspace as a set of subsets defining the topological structure of the subspace.
11. Determination of the possibility of introducing on each subspace operations of union/intersection of two elements of a set of subsets that determine the topological structure of the subspace.
12. Determination of the possibility of introducing on each subspace operations of union/intersection of any number of elements of the set of subsets that determine the topological structure of the subspace.
13. Determination of necessary conditions that must be satisfied by topological structures defined by each of the corresponding subspaces.
14. Determination of the possibility of representing IFS in the form of two disjoint subsets: a) images and their representations, b) transformation schemes.
15. Determination of the possibility of introducing states of each subset according to item 14.
16. Determination of the possibility of establishing a one-to-one correspondence between the states of subsets according to item 14.
17. Determination of the necessary conditions that determine the properties of the states introduced on each of these subsets according to item 14, for constructing phase subspaces of the IFS.

Note that the mathematical theory of image analysis may also be interested in presenting a universal model for generating DAS for image recognition in the form of a formal system.

6 Conclusion

The review and definition of problems that arise in connection with the problem of mathematical characterization of IFS determine the program and content of further research by the authors on IFS in the framework of the development of Descriptive Theory of Image Analysis.

In accordance with the models describing the architecture and structure of the recognition process described above, new IRRF schemes will be constructed.

The corresponding results will be presented in subsequent publications.

References

1. Gurevitch, I.B.: The Descriptive Framework for an Image Recognition Problem. In: Proceedings of the 6th Scandinavian Conference on Image Analysis (Oulu, June 19 - 22, 1989): in 2 volumes, **1**, 220 – 227 (1989).
2. Gurevich, I.: The Descriptive Approach to Image Analysis. Current State and Prospects. In: Heikki Kalviainen, Jussi Parkkinen, Arto Kaarna (Eds.): 14th Scandinavian Conference on Image Analysis, LNCS 3540, p.214-223, Springer-Verlag Berlin Heidelberg (2005).
3. Gurevich, I.B., Yashina, V.V.: Descriptive Approach to Image Analysis: Image Models. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **18**(4), 518-541 (2008).
4. Gurevich, I.B., Yashina, V.V.: Descriptive Approach to Image Analysis: Image Formalization Space. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **22**(4), 495-518 (2012).
5. Gurevich, I.B., Yashina, V.V.: Descriptive Image Analysis. Genesis and Current Trends. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **27**(4), 653–674 (2017).
6. Gurevich, I.B., Yashina, V.V.: Descriptive Image Analysis. Foundations and Descriptive Image Algebras. International Journal of Pattern Recognition and Artificial Intelligence 33(12), 1940018-1-1940018-25 (2019).
7. Gurevich, I.B., Yashina, V.V.: Descriptive Image Analysis: Part II. Descriptive Image Models. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **29**(4), 598–612 (2019).
8. Gurevich, I.B., Yashina, V.V.: Descriptive Image Analysis: III. Multilevel Model for Algorithms and Initial Data Combining. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **30**(3), 328–341 (2020).
9. Gurevich, I.B., Yashina, V.V.: Descriptive Image Analysis: Part IV. Information Structure for Generating Descriptive Algorithmic Schemes for Image Recognition. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **30**(4), 649–665 (2019).
10. Gurevich, I., Yashina, V.: Basic Models of Descriptive Image Analysis. In: A. Del Bimbo et al. (Eds.): ICPR 2020 Workshops, LNCS **12665**, p. 275–288, Springer Nature Switzerland AG (2021).
11. Gurevich, I. B., Yashina, V.V.: Automation of Eye Disease Diagnoses Using Descriptive Image Algebras and Boolean Algebra Methods, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **34**(2), 299–308 (2024).
12. Gurevich, I.B., Yashina, V.V. Descriptive Image Analysis: Part V. On the Characterization of the Image Formalization Space as a Mathematical Object—Goals and Problems, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications, **35** (4), 649–656 (2024).
13. Gurevich, I.B. , Yashina, V.V. Descriptive Image Analysis: Part VI. On Conceptual Tools for Describing the Image Formalization Space, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications, **36** (2) [in print].

14. Gurevich, I., Yashina, V. Implementation of Descriptive Algorithmic Schemes for Analyzing Human Fundus Images in a Multialgorithmic Hierarchical Image Analysis System. *SN COMPUT. SCI.* **7**, 369 (2026).
15. Gurevich, I.B., Yashina, V.V. (Eds.) “Image Analysis and Pattern Recognition. State of the Art in the Russian Federation”, World Scientific Publishing, 2025, 1001 p.
16. Ablameyko, S.V., Gurevich, I.B., Yashina, V.V. (Eds.). Special Issue "Leading Scientific Schools in Image Analysis, Pattern Recognition, and Related Problems of Computer Science in the USSR, the Republic of Belarus and the Russian Federation" // ‘Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications”, Pleiades Publishing, 2023 - vol. 33, No. 4, . pp. 599 – 1663.
17. Kabari, Ledisi & Nwamae, Principal Component Analysis (PCA) -An Effective Tool in Machine Learning, International Journals of Advanced Research in Computer Science and Software Engineering (May 2019).
18. Jonathon Shlens, A Tutorial on Independent Component Analysis, Google Research, https://arxiv.org/pdf/1404.2986 (2014).
19. Henry Adams, Mark Blumstein, Lara Kassab, Multidimensional Scaling on Metric Measure Spaces, https://arxiv.org/pdf/1907.01379 (2019).
20. Cai, C., Li, L., Zheng, C. (2019). Image Elementary Manifold and Its Application in Image Analysis, Communications in Computer and Information Science **1005** (2019).
21. Gurevich, I.B., Yashina, V.V. Computer-Aided Image Analysis Based on the Concepts of Invariance and Equivalence, Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **16**(4), 64-589 (2006).
22. Montesinos López, O. A. et al., Multivariate Statistical Machine Learning Methods for Genomic Prediction, https://doi.org/10.1007/978-3-030-89010-0_13
23. Morgenshtern, V., Scattering Transform, https://machinelearning.tf.fau.de/pdfs/mmmlisp19/7-scattering.pdf (2019).
24. Olshausen, B.A., Field, D.J., Sparse Coding with an Overcomplete Basis Set: A Strategy Employed by V1?, Vision Res. **37**(23), 3311-3325 (1997).
25. Hassan Hafez-Kolahi, Shohreh Kasaei, Information Bottleneck and its Applications in Deep Learning, https://arxiv.org/pdf/1904.03743 (2019).
26. Xin Wang, Hong Chen, Si’ao Tang, Zihao Wu and Wenwu Zhu, Disentangled Representation Learning, https://arxiv.org/pdf/2211.11695 (2024).
27. Gurevich, I.B., Nefyodov, A.V.: Block Diagram Representation of a 2D-AEC Algorithm with Rectangular Support Sets. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **15**(1), 187-191 (2005).
28. Kittler, J., Hatef, M., Duin, R., Matas, J.: On combining classifiers. IEEE transactions on pattern analysis and machine intelligence **20**(3), 226-239 (1998).
29. Suen, C. Y., Lam, L.: Multiple Classifier Combination Methodologies for Different Output Levels. Multiple Classifier Systems, LCNS 1857, p. 52-66 (2000).
30. Zhuravlev, Yu.I.: An Algebraic Approach to Recognition and Classification Problems. Pattern Recognition and Image Analysis: Advances in Mathematical Theory and Applications **8**, 59-100 (1998).